In dimensional variables the solution takes the form $(x < x_* = bt^\delta)$

$$v = \frac{x}{t} - \frac{(2\beta+3)(\gamma-1)}{\gamma+1}\frac{x_*}{t}, \quad v_i = \varepsilon_i^{(\beta)}\frac{x_i}{t}$$

$$\rho = \frac{A}{x_*^{\omega} t^{\beta}}\frac{\gamma+1}{\gamma-1}\left\{\frac{\gamma+1}{\delta}\left[\frac{2\beta+3}{\gamma+1} - (\beta+1)\frac{x}{x_*}\right]\right\}^{\alpha_2}$$

$$p = \frac{2\delta(\gamma-1)\rho}{\gamma+1}\left(\frac{x_*}{t}\right)^2\left[\frac{2\beta+3}{\gamma+1} - (\beta+1)\frac{x}{x_*}\right]$$

The corresponding schematic graphs of the density, pressure and velocity distribution are given in Fig.4 (the symbols accompanying the curves are the same as in Fig.3 $f = (2\beta+3)\delta^{-1}$).

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INVESTIGATION OF A THREE-DIMENSIONAL HYPERSONIC VISCOUS SHOCK LAYER ON BLUNT BODIES AROUND WHICH FLOW OCCURS AT ANGLES OF ATTACK AND SLIPPAGE*

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The three-dimensional hypersonic flow of dissociating non-equilibrium air past smooth blunt bodies with a catalytic surface is considered. An approximate numerical method of solving the equations of a hypersonic, three-dimensional viscous shock layer (SL for short) is proposed, allowing the study of flows not possessing planes of symmetry. The method is based on introducing, on the body surface, an orthogonal (x^1, x^3), coordinate system attached to the stream lines. The tangents to these stream lines are parallel to the incoming flow velocity vector component lying in the plane parallel to the body surface. The system of equations is written in this coordinate system, and derivatives with respect to the transverse coordinate x^3 of all functions sought are all omitted with exception of the pressure P and the transverse component u^3 of the velocity vector. The derivative $\partial u^3/\partial x^3$ is found from the momentum equation in the x^3 direction differentiated with respect to x^3 and simplified appropriately. The resulting system of equations is identical with the initial system near the critical point and the planes of symmetry, provided that the latter exist. Some results of computing flows at different angles of attack and slippage are given for elliptical paraboloids with various cases of catalytic reactions taking place on the body surface.

Flows past wings of infinite span, for angles of attack and slippage were investigated in /1/. Three-dimensional flows with a plane of symmetry were studied in /2—4/, a triagnular wing at large angles of attack was considered in /2/ and a body of complex shape was considered in /3, 4/.

1. Formulation of the problem.

The equation of a SL can be written, taking the chemical non-equilibrium equations and multicomponent diffusion into account and neglecting

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**Eleonor Arkad'evich Gershbein (1937-1985) was the author of a monograph on hypersonic aerodynamics and of a number of fundamental papers on the theory of boundary and shock layers, gas dynamics and heat transfer in multicomponent gas mixtures.

the baro- and thermodiffusion and diffusive thermal effect, in the form

$$\frac{\partial}{\partial x^i}\left(\rho u^i \sqrt{\frac{a}{a_{(ii)}}}\right)=0, \quad \rho A^3_{\alpha\beta}u^\alpha u^\beta = -\frac{\partial P}{\partial x^3} \tag{1.1}$$

$$\rho(Du^\gamma + A^\gamma_{\alpha\beta}u^\alpha u^\beta) = -\varepsilon a^{\gamma\alpha}\sqrt{a_{(\gamma\gamma)}}\frac{\partial P}{\partial x^\alpha} + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K}\frac{\partial u^\gamma}{\partial x^3}\right)$$

$$\rho c_p DT = 2\varepsilon D^* P + \frac{\partial}{\partial x^3}\left(\frac{\mu c_p}{\sigma K}\frac{\partial T}{\partial x^3}\right) - \sum_{k=1}^N h_k^\circ w_k^\cdot - \left(\sum_{k=1}^N c_{pk} I_k\right)\frac{\partial T}{\partial x^3} + \frac{2\mu}{K}B_{\alpha\beta}\frac{\partial u^\alpha}{\partial x^3}\frac{\partial u^\beta}{\partial x^3}$$

$$\rho Dc_i + \frac{\partial I_i}{\partial x^3} = w_i^\cdot, \quad i=1,\ldots,N-\mathrm{Ne}$$

$$\rho Dc_j^* + \frac{\partial I_j^*}{\partial x^3} = 0, \quad j=1,\ldots,\mathrm{Ne}-1$$

$$\frac{\mu}{K}\frac{\partial(mc_n)}{\partial x^3} = \sum_{t=1}^N \frac{m^2}{m_t}S_{nt}(c_n I_t - c_t I_n), \quad n=1,\ldots,N-1$$

$$P=\frac{\rho T}{m}, \quad \sum_{j=1}^{\mathrm{Ne}} I_j^* = 0, \quad \sum_{j=1}^{\mathrm{Ne}} c_j^* = 1, \quad D^* \equiv \frac{u^\alpha}{\sqrt{a_{(\alpha\alpha)}}}\frac{\partial}{\partial x^\alpha},$$

$$D \equiv D^* + u^3\frac{\partial}{\partial x^3}, \quad B_{\alpha\beta} = \frac{a_{\alpha\beta}}{\sqrt{a_{(\alpha\alpha)}a_{(\beta\beta)}}}, \quad K=\varepsilon\,\mathrm{Re}, \quad \mu_0=\mu(T_0),$$

$$T_0 = \frac{V_\infty^2}{2c_{p\infty}}, \quad \sigma=\frac{\mu c_p}{\lambda}, \quad S_{ij}=\frac{\mu}{\rho D_{ij}}, \quad \mathrm{Re}=\frac{\rho_\infty V_\infty R_0}{\mu_0},$$

$$h=\sum_{k=1}^N c_k h_k^\circ, \quad c_p=\sum_{k=1}^N c_k c_{pk}, \quad \varepsilon=\frac{\rho_\infty}{\rho_s}$$

Here and repeated indices denote summation, and no summation is performed over the indices enclosed within the round brackets; the Latin indices take the values 1, 2, 3 unless otherwise stated, and the Greek indices take the values 1, 2; $a_{\alpha\beta}$ are the coefficients of the first quadratic form of the surface; the form of the coefficients $A_{\alpha\beta}^i$, dependent on the metric will be given below; $V_\infty u^\alpha$, $\varepsilon V_\infty u^3$ are the physical coefficients of the velocity vector corresponding to the axes x^1, x^2, εx^3; $\rho_\infty V_\infty^2 P$, $\rho_\infty \rho/\varepsilon$, $T_0 T$, $c_{p\infty} T_0 h$ are, respectively, the density, temperature and enthalpy of the gas mixture consisting of N chemical components; $\mu(T_0)\mu$, λ, $c_{p\infty}c_p$, σ, m are the viscosity and thermal conductivity, specific heat capacity, Prandtl number and molecular weight of the mixture; c_j, m_j, $c_{p\infty}T_0 h_j^0$, $c_{p\infty}c_{pj}$, $\rho_\infty V_\infty I_j$, $\rho_\infty V_\infty w_j^\cdot/(\varepsilon R_0)$ are the mass concentration, molecular weight, specific enthalpy and heat capacity, normal component of the diffusion flux vector and the rate of formation of the j-th component, c_k^*, $\rho_\infty V_\infty I_k^*$ is the concentration and normal component of the diffusion flux vector of the k-th component, Ne is the number of elements, D_{ij}, S_{ij} $(i, j = 1, \ldots, N)$ are the binary diffusion coefficient and Schmidt number; $R_G = 2c_{p\infty}R$ is the universal gas constant and V_∞ is the modulus of the incoming flow velocity vector. The indices ∞, w refer to the parameters in the incoming flow and on the body surface, and the index s denotes the gas parameters behind the shock wave. All linear dimensions refer to the characteristic dimension R_0, representing one of the radii of curvature of the blunt part of the body.

In considering the chemical reactions we shall assume that we have, in the shock wave, the chemical components N_2, N, O_2, O, NO between which dissociation-recombination reactions take place of the form $A_2 + M \rightleftarrows 2A + M$ where A_2, A, M denote, respectively, the molecules N_2, O_2, atoms N, O and a third particle which may be represented by an of the five components. We also have the dissociation-recombination reaction of the form $NO + M \rightleftarrows N + O + M$ and exchange reactions $NO + O \rightleftarrows N + O_2$, $NO + N \rightleftarrows O + N_2$, $N_2 + O_2 \rightleftarrows 2NO$.

The dependence of the rate constants of the forward and reverse reactions on temperature was taken from /6/.

In computing the specific heat capacities and enthalpy of the air components, the data of /7/ were used. It was assumed that the internal degrees of freedom of the molecules were excited in the equilibrium mode. The values of the reduced collision integrals $\Omega^{(\alpha,\beta)*}(\bar{T})$ necessary to compute the transport coefficients were approximated in relation to the reduced temperature $\bar{T} = kT/\varepsilon$, using the data of /8/ for the Lenard-Jones potential. The values of the force constants were taken from /9/ and the transport coefficients were calculated using the formulas of /10, 11/.

The generalized Rankine-Hugoniot conditions are used as the boundary conditions for Eqs. (1.1) on the shock wave for $x^3 = x_s^3(x^1, x^2)$. They have the following form in the supersonic approximation in the $\{x^i\}$-coordinate system:

$$\rho\left(u^3 - \frac{u^\alpha}{\sqrt{a_{(\alpha\alpha)}}}\frac{\partial x_s^3}{\partial x^\alpha}\right) = u_\infty^3 \tag{1.2}$$

$$u_\infty^3(u^\gamma - u_\infty^\gamma) = \frac{\mu}{K}\frac{\partial u^\gamma}{\partial x^3}, \quad P = (u_\infty^3)^2$$

$$u_\infty^3(c_i - c_{i\infty}) + I_i = 0, \quad i = 1, \ldots, N - \mathrm{Ne}$$

$$u_\infty^3(c_j^* - c_{j\infty}^*) + I_j^* = 0, \quad j = 1, \ldots, \mathrm{Ne} - 1$$

$$u_\infty^3(H - H_\infty - (u_\infty^3)^2) = q + \frac{2\mu}{K}B_{\alpha\beta}u^\alpha\frac{\partial u^\beta}{\partial x^3}$$

$$q = \frac{\mu c_p}{\sigma K}\frac{\partial T}{\partial x^3} - \sum_{k=1}^{N} h_k^\circ I_k, \quad H = h^\circ + B_{\alpha\beta}u^\alpha u^\beta$$

When the boundary conditions on the body surface and given, we shall neglect the slippage rate, the temperature jumps and the concentrations of the components. The boundary conditions for the velocity components on an impermeable surface are

$$x^3 = 0, \; u^\alpha = 0, \; u^3 = 0$$

The boundary condition for determining the equilibrium temperature on the surface of the body, written ignoring the seepage of heat into the body, has the form

$$q = \Gamma T^4, \; \Gamma = 2\varepsilon\sigma_B T_0^4/(\rho_\infty V_\infty^3) \tag{1.3}$$

(ε_I are the surface blackness coefficients and σ_B is the Stefan-Boltzman constant).

When heterogeneous catalytic reactions take place on the surface of the body, we have

$$I_i = r_i^\cdot, \; i = 1, \ldots, N - \mathrm{Ne} \tag{1.4}$$

$$I_j^* = 0, \; j = 1, 2, \ldots, \mathrm{Ne} - 1$$

Here $\rho_\infty V_\infty r_i^\cdot$ is the surface rate of formation of the i-th component resulting from all heterogeneous catalytic reactions.

Two models of the first-order reaction were considered, with constant or temperature-dependent rate constants of the heterogeneous catalytic reactions:

$$r_i^\cdot = -\rho K_W c_i, \; i = \mathrm{O, N, NO} \tag{1.5}$$

Here

$$V_\infty K_{W_0} = 3 \text{ m/sec}, \quad V_\infty K_{W_N} = 1 \text{ m/sec (model I)}$$

or

$$K_{w_*} = \sqrt{\frac{RT}{2\pi m_*}}\, a_* \exp\left(-\frac{E_*}{T_k}\right), \quad a_{\mathrm{O}} = 16{,}0, \quad a_{\mathrm{N}} = 0.071$$

$* = \mathrm{O, N}$; $E_{\mathrm{O}} = 10271$ K, $E_{\mathrm{N}} = 2219$ K, where T_k is the temperature in degrees K (model II /12/). It was assumed in both models that $r_{\mathrm{NO}}^\cdot = 0$.

2. A method of solving the SL equation. Simplifying the initial formulation.

We use the system x^1, x^2, as the surface coordinate system attached to the stream lines ($x^2 = \mathrm{const}$). The tangents to these lines coincide with the components of the incoming flow velocity vector lying in the plane tangent to the body surface. The numerical and approximate analytic solutions show /13/ that at small values of Re the stream lines in the shock layer differ little from the basis lines.

Let us write (1.1) in the given (x^1, x^2) coordinate system and neglect the derivatives with respect to x^2 of all functions except P and u^2. We further introduce $w \equiv \partial u^2/\partial x^2$. We determine the latter by applying the operator $\partial/\partial x^2$ to the momentum equation projected on to x^2 and neglect the derivatives with respect to x^2 of the required functions.

As a result we obtain the following system ($h_1^2 \equiv a_{11}, h_2^2 \equiv a_{22}$):

$$\frac{\partial}{\partial x^1}(\rho u^1 h_2) + \rho h_1 w + \frac{\partial}{\partial x^3}(\rho u^3 h_1 h_2) = 0 \tag{2.1}$$

$$\rho\left(\frac{u^1}{h^1}\frac{\partial u^1}{\partial x^1} + u^3\frac{\partial u^1}{\partial x^3} + A_{\alpha\beta}^1 u^\alpha u^\beta\right) = -\frac{\varepsilon}{h_1}\frac{\partial P}{\partial x^1} + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K}\frac{\partial u^1}{\partial x^3}\right)$$

$$\rho\left(\frac{u^1}{h_1}\frac{\partial u^2}{\partial x^1} + u^3\frac{\partial u^2}{\partial x^3} + A_{\alpha\beta}^2 u^\alpha u^\beta\right) = -\frac{\varepsilon}{h_2}\frac{\partial P}{\partial x^2} + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K}\frac{\partial u^2}{\partial x^3}\right)$$

$$\rho\left(\frac{u^1}{h^1}\frac{\partial w}{\partial x^1} + u^3\frac{\partial w}{\partial x^3} + \frac{\partial A_{11}^2}{\partial x^2}(u^1)^2 + 2A_{12}^2 u^1 w + \frac{w^2}{h_2}\right) =$$

$$-\varepsilon\frac{\partial}{\partial x^2}\left(\frac{1}{h_2}\frac{\partial P}{\partial x^2}\right) + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K}\frac{\partial w}{\partial x^3}\right)$$

$$\partial P/\partial x^3 = -\rho A_{\alpha\beta}^3 u^\alpha u^\beta$$

$$\rho c_p\left(\frac{u^1}{h^1}\frac{\partial T}{\partial x^1}+u^3\frac{\partial T}{\partial x^3}\right)=2\varepsilon\left(\frac{u^1}{h_1}\frac{\partial P}{\partial x^1}+\frac{u^2}{h^2}\frac{\partial P}{\partial x^2}\right)+$$
$$\frac{\partial}{\partial x^3}\left(\frac{\mu c_p}{\sigma K}\frac{\partial T}{\partial x^3}\right)-\sum_{k=1}^{N}h_k{}^\circ w_k{}^\cdot-\left(\sum_{k=1}^{N}c_{pk}I_k\right)\frac{\partial T}{\partial x^3}+$$
$$\frac{2\mu}{K}B_{\alpha\beta}\frac{\partial u^\alpha}{\partial x^3}\frac{\partial u^\beta}{\partial x^3}$$
$$\rho\left(\frac{u^1}{h_1}\frac{\partial c_i}{\partial x^1}+u^3\frac{\partial c_i}{\partial x^3}\right)+\frac{\partial I_i}{\partial x^3}=w_i^\cdot,\quad i=N-\mathrm{Ne},\ldots,N$$
$$\rho\left(\frac{u^1}{h_1}\frac{\partial c_j^*}{\partial x^1}+u^3\frac{\partial c_j^*}{\partial x^3}\right)+\frac{\partial I_j^*}{\partial x^3}=0,\quad j=1,\ldots,\mathrm{Ne}$$

In writing down system (2.1) we used the orthogonality of x^1, x^2 on the surface.

The boundary conditions for system (2.1) remain as before ((1.2)−(1.5)), and the boundary conditions for w are obtained by differentiating (1.2) with respect to x

$$w=0,\quad x^3=0;\quad w=\frac{\mu}{Ku_\infty{}^3}\frac{\partial w}{\partial x^3},\quad x^3=x_s{}^3(x^1,x^2) \tag{2.2}$$

Here we assumed, by virtue of the choice of x^1 and x^2 that $u_\infty{}^2=0$. We can obtain $\partial P/\partial x^2$ and $\partial^2P/(\partial x^2)^2$ using the fifth equation of system (2.1) differentiated with respect to x^2 and simplified in a similar manner. However, to simplify the problem we have used Newton's formula to specify the pressure gradient components $\partial P/\partial x^2$.

It should be noted that in boundary-layer theory the method of axisymmetric analogy is widely used. In this method the equations are written in a coordinate system attached to the stream lines of the inviscid flow, and terms containing u^2 and $\partial/\partial x^2$ are omitted. The greatest error of the method can occur near the critical point, near the line of flow $(u^2=0,\ \partial u^2/\partial x^2=0\,(1))$ etc. Unlike the method of axisymmetric analogy, the present method, which can be used to solve the equations of three-dimensional boundary layer, becomes exact near the plane of symmetry of the flow (provided that $\partial^2P/(\partial x^2)^2$ is given by Newton's formula).

Calculation of the metric coefficients. The coefficients $A_{\alpha\beta}{}^i$ have the following form in the x^1, x^2 coordinate system orthogonal to the body surface /5/:

$$A^\beta_{\alpha\alpha}=-2A_{12}{}^\alpha=-\frac{1}{h_1h_2}\frac{\partial h_\alpha}{\partial x^\beta}(\beta\neq\alpha),\quad A^\alpha_{(\alpha\alpha)}=0,\quad A^3_{\alpha\beta}=\frac{b_{\alpha\beta}}{h_{(\alpha)}h_{(\beta)}}$$

We find h_α, $\partial h_\alpha/\partial x^\beta$, $b_{\alpha\beta}$ as follows. Let the surface S be given in a Cartesian coordinate system by the equation $z^3=f(z^1,z^2)$. Parametrizing the surface S in the usual manner $y^\alpha=z^\alpha$, $z^3=f(y^1,y^2)$ (the y^3 axis is directed along the normal to the body), we obtain the coefficients of the first and second quadratic form (in the y^1, y^2, y^3 system)

$$a_{\alpha\alpha}=1+(g_\alpha)^2,\quad a_{12}=g_1g_2,\quad b_{\alpha\beta}=-g_{\alpha\beta}/\sqrt{a} \tag{2.3}$$
$$a=\det\|a_{\alpha\beta}\|,\quad g_\alpha=\partial f/\partial y^\alpha,\quad g_{\alpha\beta}=\partial^2f/\partial y^\alpha\partial y^\beta$$

Let the unit vector of the incoming flow in the system z^1, z^2, z^3 have the form $\mathbf{V}_\infty=V_0{}^i\mathbf{e}_i$, $V_0{}^iV_0{}^i=1$. Then its components $u_\infty{}^1$, $u_\infty{}^2$, $u_\infty{}^3$ in the system y^1, y^2, y^3 will be

$$au_\infty{}^1=(1+g_2{}^2)V_0{}^1-g_1g_2V_0{}^2+g_1V_0{}^3\equiv V^1$$
$$au_\infty{}^2=(1+g_1{}^2)V_0{}^2-g_1g_2V_0{}^1+g_2V_0{}^3\equiv V^2$$
$$\sqrt{a}\,u_\infty{}^3=g_1V_0{}^1+g_2V_0{}^2-V_0{}^3$$

The vector orthogonal on the surface to the vector $(u_\infty{}^1,u_\infty{}^2)$, has the components $(U^1/\sqrt{a}$, $U^2/\sqrt{a})$

$$U^1\equiv\pm(g_2V_0{}^3+V_0{}^2),\ U^2\equiv\mp(g_1V_0{}^3+V_0{}^1)$$

Let x^1, x^2 be the system introduced in Sect.2. Then

$$\left.\frac{\partial y^\alpha}{\partial x^1}\right|_{x^2}=\frac{h_1}{\sqrt{aV_t}}V^\alpha,\quad \left.\frac{\partial y^\alpha}{\partial x^2}\right|_{x^1}=\frac{h_2}{\sqrt{V_t}}U^\alpha \tag{2.4}$$
$$V_t\equiv(g_1V_0{}^3+V_0{}^1)^2+(g_2V_0{}^2+V_0{}^1)^2+(g_2V_0{}^1-g_1V_0{}^2)^2$$

To find h_2 we will use a method similar to that in /14/. We assume that the functions $y^\alpha(x^1,x^2)$ are sufficiently smooth and, that inverse functions $x^\alpha(y^1,y^2)$ exist. Let

$$G\equiv\left.\frac{\partial y^2}{\partial y^1}\right|_{x^2}=\frac{V^2}{V^1} \tag{2.5}$$
$$F\equiv\left.\frac{\partial y^2}{\partial x^2}\right|_{y^1}=\left.\frac{\partial y^2}{\partial x^1}\right|_{x^2}\left.\frac{\partial x^1}{\partial x^2}\right|_{y^1}+\left.\frac{\partial y^2}{\partial x^2}\right|_{x^1}=h_2\frac{U^2V^1-U^1V^2}{V_t^{1/2}V^1}=h_2\frac{V_t^{1/2}}{V^1} \tag{2.6}$$

In (2.6) we have used (2.4). Assuming that $\partial^2 y^2/\partial y^1 \partial x^2$ and $\partial^2 y^2/\partial x^2 \partial y^1$ are continuous, we obtain

$$\frac{\partial F}{\partial y^1} + G\frac{\partial F}{\partial y^2} = F\frac{\partial G}{\partial y^2} \tag{2.7}$$

Substituting (2.5) into (2.6) and (2.7), we obtain

$$V^\alpha \frac{\partial h_2^*}{\partial y^\alpha} - h_2^*\left(\frac{\partial V^\alpha}{\partial y^\alpha}\right) = 0, \quad h_2^* = h_2 V_t^{1/2} \tag{2.8}$$

Let us construct the solution of (2.8). Let $y^1(s,\tau)$, $y^2(s,\tau)$ be a solution of the set of equations for determining the stream lines

$$dy^\alpha/ds = V^\alpha(y^1, y^2), \quad y^\alpha(0,\tau) = y_0^\alpha(\tau)$$

Then

$$h_2(s,\tau) = \frac{h_{20}^*(\tau)}{V_t^{1/2}} \exp \int_0^s \left(\frac{\partial V^\alpha}{\partial y^\alpha}\right)\bigg|_{y^\alpha = y^\alpha(s^1,\tau)} ds^1$$

$$h_2^*(0,\tau) = h_0^*(\tau), \quad ds/dx^1 = h_1/(aV_t^{1/2})$$

We can find h_1 is the same manner, although we shall do it differently. Since

$$\frac{\partial y^2}{\partial y^1}\bigg|_{x^1} = -\frac{(q_1 V_0^3 + V_0^1)}{q_2 V_0^3 + V_0^2} \tag{2.9}$$

it follows that the first integral of (2.9) had the form

$$fV_0^3 + V_0^2 y^2 + V_0^1 y^1 = g(x^1)$$

Choosing $g(x^1) = (x^1)^2/2$, we obtain

$$(x^1)^2 = 2(V_0^1 y^1 + V^2 y^2 + fV_0^3) \tag{2.10}$$

$$g^{11}(x^1, x^2) = a^{\alpha\beta}(y^1, y^2)\frac{\partial x^1}{\partial y^\alpha}\frac{\partial x^1}{\partial y^\beta} = \frac{V_t}{a(x^1)^2}$$

where we have used relations (2.3). Then

$$h' = (g^{11})^{-1/2} = x^1 a^{1/2} V_t^{-1/2}$$

Now we can find all the coefficients $A_{\alpha\beta}{}^i$ as functions of y^1, y^2, using the formulas for passing from one coordinate system to another

$$b^1_{\alpha\beta}(x^1, x^2) = b_{\lambda\gamma}(y^1, y^2)\frac{\partial y^\lambda}{\partial x^\alpha}\frac{\partial y^\gamma}{\partial x^\beta}$$

and calculating the derivatives from the formulas

$$\frac{\partial}{\partial x^\alpha} = \frac{\partial y^\beta}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}$$

where $\partial y^\beta/\partial x^\alpha$ are obtained from (2.4).

3. Numerical solution.

To obtain the numerical solution, we have introduced new Dorodnitsyn-type variables

$$\xi^\alpha = x^\alpha, \quad \zeta = \frac{1}{\Delta}\int_0^{x^3} \rho h_1 h_2 \, dx^3, \quad \Delta = \int_0^{x_\delta^3} \rho h_1 h_2 \, dx^3 \tag{3.1}$$

and new unknown functions by means of the formulas

$$u^1 = u_*^1(\xi^1, \xi^2)\frac{\partial \varphi_1}{\partial \zeta}, \quad u^2 = u_*^2(\xi^1, \xi^2)\frac{\partial \varphi_2}{\partial \zeta}, \quad w = w_*(\xi^1, \xi^2)\frac{\partial \varphi_w}{\partial \zeta}$$

$$\rho h_1 h_2 u^3 = -\frac{\partial}{\partial x^1}(\Delta \varphi_1^* \varphi_1) - \Delta \varphi_w^* \varphi_w, \quad \varphi_\alpha^* = \frac{u_*^\alpha}{h_\alpha}, \quad \varphi_w^* = \frac{w_*}{h_2}$$

$$T = T^*(\xi^1, \xi^2)\,\theta, \; u_*^1 = x^1, \; u_*^2 = x^1, \; w^* = h_2, \; T^* = (u_\infty^3)^2$$

$$X_i = \frac{h_1 h_2}{\Delta} I_i, \quad i = 1, \ldots, \mathrm{N} - \mathrm{Ne};$$

$$X_j^* = \frac{h_1 h_2}{\Delta} I_j^*; \quad j = 1, \ldots, \mathrm{Ne}$$

The resulting system was solved using the method given in /1/.

We have taken, as initial conditions, the solution of the SL equations near the critical point, written in the coordinate system associated with the principal curvatures. Next the streamlines were found, the metric coefficients and coefficients of the equations are calculated, and SL was solved along each fixed streamline.

As an example we considered flows past elliptical paraboloids whose equation in a Cartesian coordinate system attached to the apex was $2z^3 = (z^1)^2 + k(z^2)^2$ where $k = R_1/R_2$, $R_1 = R_0 = 0.5$ m. The unit velocity vector of the incoming flow was given in terms of the components $\{V_0{}^i\}$ in a Cartesian coordinate system. The parameters of the incoming flow were as follows: $\rho_\infty = 5.9\cdot10^{-8}$ g/cm^3, $V_\infty = 7.25$ km/sec. and the blackness coefficient was $\varepsilon_I = 0.85$.

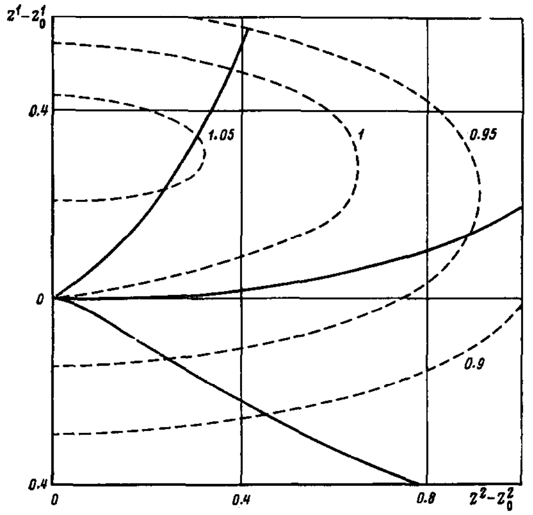

Fig.1

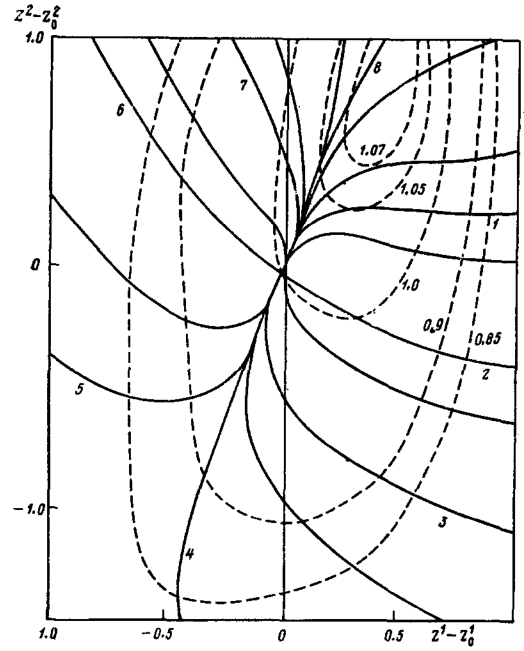

Fig.2

Fig.1 shows the streamlines $x^3 = \mathrm{const}$ (the solid lines) computed for the case of flow past an elliptical paraboloid $k = 0.4$ at an angle of attack equal to 45°, $V_0 = (\sqrt{2}/2, 0, \sqrt{2}/2)$ (here and henceforth $z_0{}^1, z_0{}^2$ are the coordinates of the critical point). The figure also shows the isolines of thermal fluxes q_r (the dashed lines, the number accompanying each isoline is equal to the thermal flux relative to the thermal flux q_0 at the critical point $q_0 = 34.5$ wt./cm^2) calculated for model II of the catalytic activity of the surface. For some of the streamlines the maximum of the thermal flux is displaced in relation to the critical point, and the absolute maximum lies in the plane of symmetry. Thus we have a whole region bounded by the isoline $q_r = 1.0$, in which the thermal flux is greater than the flux at the critical point.

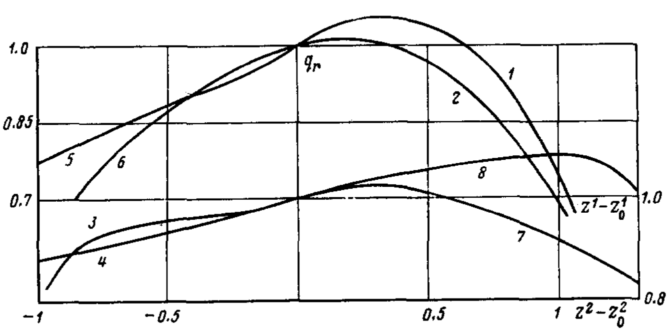

Fig.3

Fig.2 shows the streamlines $x^3 = \mathrm{const}$ (the solid lines) and the isolines of thermal fluxes q_r (dashed lines) of the case of flow past an elliptic paraboloid $k = 0.5$ at the angle of attack and slippage, $V_0 = (\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$ for model I of heterogeneous catalytic reactions. Here we also have a region with thermal fluxes greater than the flux at the critical point ($q_0 = 21.3$ wt./cm^2).

Fig.3 shows the distribution of q_r along some streamlines, with the curves 1–8 corresponding to the curves 1–8 of Fig.2.

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EFFECTS OF LOCALIZATION AND FORMATION OF STRUCTURES DURING THE COMPRESSION OF A FINITE MASS OF GAS IN A PEAKING MODE*

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The problem of the adiabatic compression of a finite mass of gas with a cylindrical or spherical piston is considered. The pressure at the piston increases in the peaking mode according to the law $P(0,t) = P_0(t_{foc} - t)^{n_S}$, $n_S = -2\gamma(N+1)/(\gamma+1+N(\gamma-1))$, i.e. it becomes infinite as $t \to t_{foc}$, $N = 0, 1, 2$ is the symmetry index and γ is the adiabatic index. The entropy of the gas is distributed over the Lagrangian mass coordinate m: $s = \ln\{a_0|m - m_1|^\delta\}$, a_0, m_1, δ are parameters. The existence of localization of hydrodynamic processes is shown for the case when $N=0$; in spite of the unlimited growth of pressure at the piston the perturbations do not penetrate beyond a certain finite mass of gas (the region of localization). Outside the region of localization the gas is not affected by the piston and remains in its initial state. The other effect consists of the formation (when $\delta \neq 0$) of gas-dynamic structures, including complex ones such as localized temperature or density maxima connected with the fixed mass of gas.

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